A Different Angle: New Tools For Circular Data

Scientific background

Circular data are data measured in the form of angles or two-dimensional orientations and arise commonly almost everywhere throughout science. Two principal circular measuring instruments are the compass (0º-360º) and the clock (0-24 hour). To give just a few examples consider, for instance, wind and ocean current directions, directions of birds or animals from points of release, and times of day of accident occurrences (Fisher, 1995). The typical examples are usually from biology, meteorology, astronomy, earth sciences, and medicine. However, also in social and behavioural research circular data are regularly encountered as will be illustrated by a selection of examples from published research in this area:

- An experiment on bimanual circling, as pictured in Figure 1, investigated movement control and the role of visual feedback with as measure of performance the difference in angular degrees between the left and right hand (Mechsner et al., 2007). The importance of studies on periodic bimanual movement for our understanding of the basic organizational principles of human actions is summarized in a *Nature* article by Mechsner et al. (2001). Other examples of similar studies can be found in Kelso (1984), Johnson (1998), Swinnen et al. (1998), Spencer et al. (2005), Postma et al. (2008), and many more.
- In cognitive psychology, various experiments are performed using a moving room as presented in Figure 2 (Lee & Aronson, 1974), or modern, virtual reality versions of it (see Stoffregen et al., 2004). With these experiments, the effect of vision on postural control is investigated. Oullier et al. (2002), for instance, measured postural coordination in looking and tracking tasks as angular displacement of ankle and hip. Other moving room examples are, among others, Smart et al. (2002), Bardy et al. (2002), Dijkstra et al. (1994).
- In educational research, a common circular mapping instrument is the Teacher Interpersonal Circle (see Figure 3), which is an application of Leary's circle (Leary, 1957). Inter-personal traits and behaviour are measured as angular positions on the circumference of the scale. Brekelmans and colleagues (technical report), for instance, used this circumplex instrument in a longitudinal study for the investigation of the effect of interpersonal traits and behaviour of teachers on student motivation. Another example of a circumplex model is Russel's mood scale (Russel, 1980) that locates specific moods in the circumplex perimeter with a corresponding angle.

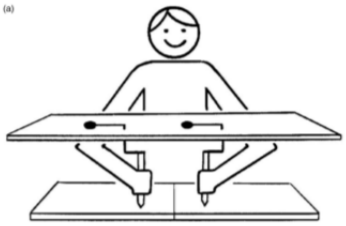

Fig. 1: Apparatus for bimanual circling

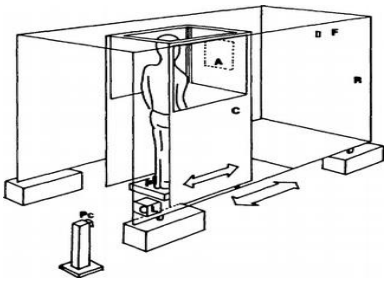

Fig. 2: Moving room set-up

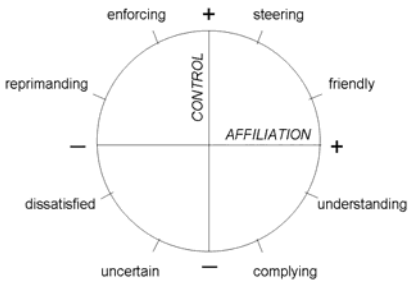

Fig. 3: Teacher Interpersonal Circle

Many more applications can be found in other areas of social, behavioural and related sciences, e.g., in social neurosciences (Aarts et al., 2012), developmental psychology (Bullens et al., 2010a,b), criminology (Brunsdon & Corcoran, 2006), demography (Coleman & Haskey, 1986), social psychology (Kubiak & Jonas, 2007), and political sciences (Haskey, 1988).

For the analysis of circular data, special directional statistical methods are required which take into account the circular structure of the sample space. This is easily illustrated by considering the computation of the mean of two directional observations, for instance, 350° and 10°. The arithmetic mean is (350+10)/2=180, but the mean direction is 0° or, equivalently, 360°. Furthermore, for inferential statistics the normality assumption that is common for linear data does not apply for circular data. The natural analogue on the circle of the normal distribution on the real line is the von Mises distribution (Fisher, 1995; Mardia and Jupp, 2000) and available tests for circular data are often based on this distribution. For instance, to test the equality of several independent group mean directions, a modified version of the analysis of variance (ANOVA) is available, that is designed to deal with the directional nature of the data and makes use of a von Mises likelihood function (e.g. Stergiou, 2004; Jammalamadaka & Sengupta, 2001; Fisher, 1995).

The field of circular statistics is, however, much less developed than the field of linear statistics. This is also noticed by empirical researchers dealing with directional data in experimental settings, for instance, by Smart et al. (2002), who investigated motion sickness and the role of postural instability. Their research is another example of a moving room experiment. Participants were exposed to an optical simulation of body sway. The effects of vision (eyes open/closed) and group (sick/well during experiment) on several outcome measures reflecting body sway (e.g. postural motion and vection) were investigated. For the linear outcome variables a mixed ANOVA with group as between factor and vision as within factor was applied and main and interaction effects tested and interpreted. For the directional outcome 'phase' (measured in degrees), the researchers applied the circular analogue of a simple ANOVA to test for group differences in each visual condition separately and commented on the limited availability of tests in this area of statistical analyses: "With circular statistics, variance can be analyzed for only a single factor in each test (i.e., it is not possible to test for interactions)" (Smart et al., 2002, page 458). Although some new developments were made since the publication of the study by Smart (see also the section Methodology and references therein), the circular statistics toolbox is still limited.

From the few available publications on statistical methods for longitudinal circular data it becomes immediately clear that this, too, is a timely topic; the contributions are mainly from the last decade (e.g., D'Elia, 2001; Rocchi, 2001; Nuñez-Antonio & Gutiérrez-Peña, 2005; Bhattacharya & SenGupta, 2009; Nuñez-Antonio et al., 2011) and did not yet find their way to the social and behavioural sciences. Also in the context of longitudinal circular data the types of research questions and observed data structures are much more varied than the statistical tools that are available for the evaluation. For instance, collaborators from the educational sciences department recently needed statistical tools for data measured repeatedly over time and including both linear and circular variables. Their research would greatly benefit from the theoretical development of methods for such data, accompanied by accessible papers and user friendly software.

Overall aim

The main objective is to enable better use of data sets that contain circular variables. Such data are present in several areas of social and behavioural (and other) sciences but currently not optimally analyzed due to either a lack of appropriate statistical tools, or due to ignorance of tools that are developed in other disciplines but not easily accessible. In this project, Bayesian methods for circular data in experimental, cross-sectional and longitudinal designs will be developed and presented to potential users in a non-technical way.

Currently, researchers with circular data often turn to one of two unsatisfactory solutions: i) the periodical nature of the data is ignored without knowing the impact this may have on the validity of conclusions, and ii) the research questions are simplified to be able to evaluate them with the limited set of tools that are known and available to the researcher. With this project, new tools for circular data will be developed and presented to applied researchers such that they will be able to properly analyze the research questions that they have.